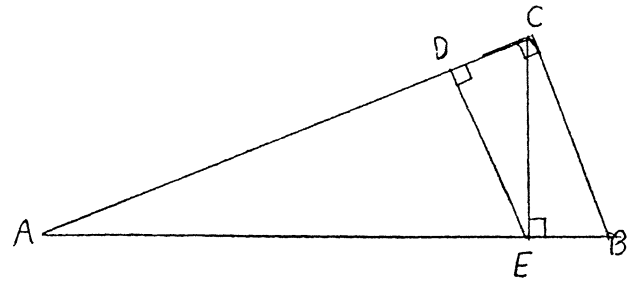
February 3, 1999 WOCOMAL VARSITY MEET

ROUND I: Similarity and Pythagorean relationships

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM OR AS DECIMALS WITH THREE PLACES TO THE RIGHT OF THE DECIMAL POINT

1. The lengths of the sides of a triangle are 9, 15, and 18. What are the lengths of the sides of a similar triangle with area 1/9 that of the given triangle?

2. If EB = 4 and AE = 16, find AC.



3. A triangle has sides of length 30, 70, and 80. The shortest altitude of the triangle divides one of the sides into two segments. Find the length of the longer of these two segments.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

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ROUND II: Algebra 1 - open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Solve: $7 - 5(3x - 2) = 5 - 7(3x - 2)$

2. A bicycle shop sells both bicycles(2 wheels) and tricycles(3 wheels). Recently the owner counted 153 wheels and 136 pedals. How many bicycles and tricycles did she have?

3. Given three positive integers a, b, and c, that satisfy both $2a + 3b + 4c = 25$ and $4a + 3b + 2c = 35$. Find all such ordered triples (a,b,c).

ANSWERS

(1 pt) 1. $X =$ _____

(2 pts) 2. _____ bicycles and _____ tricycles

(3 pts) 3. _____

Auburn, Leicester, South

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ROUND III: Functions

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. The area of a rectangle is 64 sq in. Express the perimeter p as a function of the width w.

2. For all real x let function f be defined by $f(x) = 5 - f(x-1)$. Express $f(x-2)$ in terms of $f(x-1)$.

3. Given that $f(x) = ax + 3$ and $f^{-1}(x) = 2x + b$, find the ordered pair (a,b).

ANSWERS

(1 pt) 1. $p =$ _____

(2 pts) 2. $f(x-2) =$ _____

(3 pts) 3. (,) _____

Auburn, Bartlett, Mass. Academy

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ROUND IV: Combinatorics

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Threre are 41 marbles in a bag. 8 are blue, 9 are white, 11 are purple, and 13 are red. If you pick one at a time, without looking, and don't return it, how many times must you pick to be sure of getting 3 of the same color?

2. In a certain golf tournament, each match groups three people together, so that one person wins and goes to another match in the next round, while the other two lose and are each eliminated. The tournament continues until only one person remains undefeated. If 243 players enter the tournament, how many matches must be played?

3. Find the number of positive four digit integers with no repeated digits in which the last digit is twice the first digit.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

Auburn, Mass. Academy, St.John's

ROUND V: Analytic geometry of lines and conic sections

ALL ANSWERS MUST BE IN THE FORM SPECIFIED AND INVOLVE INTEGERS, REDUCED FRACTIONS, EXACT DECIMALS, OR SIMPLIFIED RADICALS. NO DECIMAL APPROXIMATIONS.

1. Find the ordered pair of real numbers (x.y) which does satisfy $y = 5x - 15$, but does not satisfy $\frac{y}{x-3} = 5$.

2. Find the eccentricity of the ellipse with equation $\frac{x^2}{36} + \frac{y^2}{16} = 1$.

3. The equations of the asymptotes of a hyperbola are 5x - 3y = 19 and 5x + 3y = 1. If the coordinates of one of the vertices are (2,2), find the coordinates of the other vertex.

ANSWERS

(1 pt) 1. (,)

(2 pts) 2. _____

(3 pts) 3. (,)

Leicester, Mass. Academy, South

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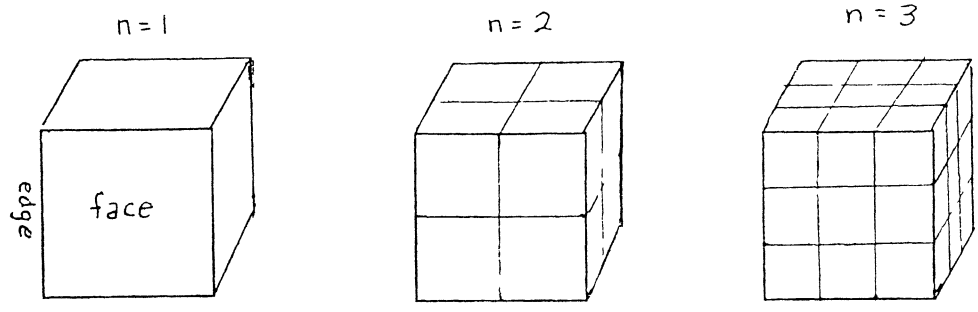
TEAM ROUND: Related problem solving

EACH ANSWER MUST BE IN THE FORM OF A SINGLE POSITIVE INTEGER

A wooden cube with five red faces and one blue face is to be divided into smaller cubes. These will have various red, blue, or plain wood faces. Let n = the number of smaller cubes per edge. Fill in the chart below and transfer to the one team answer sheet only the two thickly outlined rows. Each of the 18 answers is worth 1 point.



n	total # of smaller cubes	Number of smaller cubes with exactly ..... faces: 0 red 0 blue	1 red 0 blue	0 red 1 blue	1 red 1 blue	2 red 1 blue	1 red 2 blue	2 red 0 blue	3 red 0 blue
2									
3									
4									
5									
Sum of 4 rows above									
n = 100									

Auburn, QSC

February 2, 1999 — REGIONAL VARSITY MEET ANSWERS

ROUND I (sir Pythag)

1. 1 pt — 3, 5, 6 (any order)
2. 2 pts — $8\sqrt{5}$ or 17.888 or 17.889
3. 3 pts — 65

ROUND II (alg 1)

1. 1 pt — $\frac{1}{3}$ or $.\overline{3}$
2. 2 pts — 51 ... 17 (need both)
3. 3 pts — (6,3,1), (7,1,2) (need both; order does matter)

ROUND III (funct)

1. 1 pt — $2w + \frac{128}{w}$ (or alg equiv)
2. 2 pts — $5 - f(x-1)$
3. 3 pts — $\left(\frac{1}{2}, -6\right)$ (.5 OK)

ROUND IV (comb)

1. 1 pt — 9
2. 2 pts — 121
3. 3 pts — 224

ROUND V (analyt)

1. 1 pt — (3,0)
2. 2 pts — $\frac{\sqrt{5}}{3}$ (no decimals)
3. 3 pts — (2, −8)

TEA ROUND — 1 pt each

total #	Sum	# red, # blue	
1,000,000	224		
941,192	36	0, 0	
48,020	70	1, 0	
9,604	14	0, 1	
392	24	1, 1	
4	16	2, 1	
0	0	1, 2	
784	48	2, 0	
4	16	3, 0	